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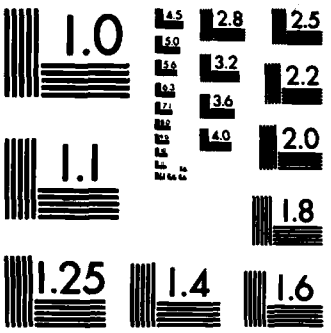

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A Class of Multivariate New Better

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by

Emad El-Neweihi*

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A Class of Multivariate New Better Than used Processes

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December, 1982.

Department of Mathematics, Statistics
and Computer Science
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20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

A multivariate version of the univariate new better than used (NBU) processes, due to El-Neweihi, Proschan and Sethuraman is introduced. Closure properties of this new class of multivariate NBU (MNBU) processes under various reliability operations are obtained. A relationship between the MNBU processes and a well known class of multivariate NBU distributions is described.

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Abstract

A multivariate version of the univariate new better than used (NBU) processes, due to El-Neweihi, Proschan, and Sethuraman is introduced. Closure properties of this new class of multivariate NBU (MNBU) processes under various reliability operations are obtained. A relationship between the MNBU processes and a well known class of multivariate NBU distributions is described.

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1. Introduction.

The theory of multistate coherent systems has motivated several authors to introduce and study special classes of stochastic processes which generalize the well known classes of lifetimes in the two-state reliability theory. In a paper by El-Neweihi, Proschan and Sethuraman (1978) a univariate class of new better than used (NBU) processes was introduced. A member in this class can serve as a model to describe the degradation of a multistate component. A multivariate process whose univariate marginals are independent NBU processes can then be used to describe the joint stochastic behavior of a collection of degradable components. However such a model is not adequate when it is unreasonable to assume stochastic independence. The main purpose of this paper is to introduce a more general class of multivariate NBU processes which extends the univariate NBU class due to El_Neweihi, Proschan and Sethuraman (1978). The approach used in this paper is similar to the one used by Block and Savits (1981) to introduce a class of multivariate IFRA processes.

A brief summary of the contents of this paper is now given. In section 2 the basic notation, terminology and definitions are given. In section 3 a class of multivariate NBU processes (hereafter referred to as the MNBU class) is defined. Necessary and sufficient conditions for a multivariate process to be a member in the MNBU class are given. Finally in section 4 it is shown that the MNBU class enjoys a number of closure properties which include as special cases some well known preservation properties of the univariate class of NBU lifetimes. Some of these properties are utilized to construct examples of MNBU processes. Using one of the characterizations established in section 3, the MNBU class is shown to be closely related to a well known class of multivariate NBU distributions.

2. Notation, Definitions and Terminology.

The vector $\underline{x} = (x_1,\ldots,x_n)$ denotes a point in R^n, the real n-dimensional Euclidean space equipped with its usual metric .

$\underline{y} \leq \underline{x}$ means that $y_i \leq x_i$, $i = 1,\ldots,n$

$\underline{y} < \underline{x}$ means that $y_i < x_i$, $i = 1,\ldots,n$

A subset $U \subseteq R^n$ is an upper set if $\underline{x} \in U$ and $\underline{x} \leq \underline{y}$ imply that $\underline{y} \in U$.

Given a vector $\underline{x} \in R^n$, $Q_{\underline{x}}$ denotes the set $\{\underline{y}: \underline{x} < \underline{y}\}$.

A function $f: R^n \to R^m$ is a nondecreasing function if $\underline{x} \leq \underline{y}$ implies that $f(\underline{x}) \leq f(\underline{y})$. A nonincreasing function is similarly defined.

Throughout the remainder of this paper all the random variables (random vectors) considered are assumed to be nonnegative.

3. The MNBU Class: Definition and Characterizations.

First let us recall that a nonnegative random variable T is said to be NBU if

$$P(T > t) \leq P(T > \alpha t)P(T > (1-\alpha)t), \tag{3.1}$$

for every $t \geq 0$ and every $0 \leq \alpha \leq 1$.

In the context of reliability theory the random variable T usually represents the lifetime (functioning time) of a component that can be in either of two states "functioning" (denoted by 1) and "failed" (denoted by 0). In a paper by El-Neweihi, Proschan and Sethuraman (1978) a class of univariate NBU stochastic process was defined as follows: A nonnegative, nonincreasing and right-continuous stochastic process $\{X(t): t \geq 0\}$

whose state space is $\{0,1,\ldots,M\}$ is said to be NBU if

$$T_j = \inf \{t: X(t) \notin \{j+1,\ldots,M\}\} \tag{3.2}$$

is NBU random variable for every $j \in \{0,1,\ldots,M\}$. Such a process was introduced to describe the stochastic behavior of a multistate component which starts at time 0 in state M (perfect functioning) and deteriorates, as time passes, to lower states until finally state 0 (complete failure) is reached. The definition of the NBU process can be easily extended to the case in which the state space S is any subset of $[0,\infty)$ by simply requiring that the random variable

$$T_a = \inf \{t: X(t) \notin (a,\infty)\} \tag{3.3}$$

is NBU for every $a \geq 0$.

A natural multivariate extension of (3.3) is now used to define the MNBU class of stochastic processes.

Definition 3.1. A vector-valued stochastic process $\{\underline{X}(t)=(X_1(t),\ldots,X_n(t)): t \geq 0\}$ is said to be a MNBU process if it is nonnegative, nonincreasing, right-continuous and the random variable

$$T_U = \inf \{t: \underline{X}(t) \notin U\} \tag{3.4}$$

in NBU for every open upper set $U \subseteq R^n$.

The class of all MNBU processes (of all dimensions) is called the MNBU class. Clearly the MNBU class includes, when n=1, the univariate NBU class due to El-Neweihi, Proschan and Sethuraman (1978). Also in a recent paper by Block and Savits (1981), the authors define a multivariate class of IFRA processes by requiring the random variable in (3.4) to be

IFRA. Clearly their class is a subclass of the MNBU class.

Remark 3.2. In view of (3.1) and the right-continuity of the process $\{\underline{X}(t): t \geq 0\}$, condition (3.4) is equivalent to

$$P(\underline{X}(t) \in U) \leq P(\underline{X}(\alpha t) \in U)P(\underline{X}((1-\alpha)t) \in U), \tag{3.5}$$

for every $t \geq 0$, $0 \leq \alpha \leq 1$ and every open upper set $U \subseteq R^n$.

In the remainder of this section two characterizations of the MNBU class are given. These characterizations are used in section 4 to establish properties of the MNBU class and relate it to a well known class of multivariate NBU distributions. First we need the following lemma whose proof is essentially due to Esary, Proschan and Walkup (1967)(see also Marshall and Shaked (1982)).

Lemma 3.3. Let $\{\underline{X}(t): t \geq 0\}$ be a stochastic process. Then condition (3.5) is equivalent to

$$P(\underline{X}(t) \in U) \leq p(\underline{X}(\alpha t) \in U)P(\underline{X}((1-\alpha)t) \in U) \tag{3.6}$$

for every $t \geq 0$, $0 \leq \alpha \leq 1$ and every upper Borel set $U \subseteq R^n$.

The following theorem gives a necessary and sufficient condition for a nonnegative, nonincreasing and right-continuous stochastic process $\{\underline{X}(t): t \geq 0\}$ to be a MNBU process.

Theorem 3.4. The process $\{\underline{X}(t): t \geq 0\}$ is a MNBU process if and only if

$$E[\phi(\underline{X}(t))] \leq E[\phi^{\alpha}(\underline{X}(\alpha t))]\ E[\phi^{1-\alpha}(\underline{X}((1-\alpha)t))], \tag{3.7}$$

for every nonnegative, nondecreasing and Borel measurable function $\phi: R^n \to R$ and every $t \geq 0$, $0 \leq \alpha \leq 1$.

Proof. Obviously (3.7) implies (3.5) by taking $\phi = I_U$, where $U \subseteq R^n$ is an open upper set. Now assume $\{\underline{X}(t): t \geq 0\}$ is a MNBU process. In view of (3.6), condition (3.7) is satisfied for all $\phi = aI_U$, where $a \geq 0$ and $U \subseteq R^n$ is an upper Borel set. Let $\phi = \sum_{i=1}^{m} \phi_i$, where $\phi_i = a_i I_{U_i}$, $a_i \geq 0$, $U_i \subseteq R^n$ is an upper Borel set, $i = 1,\ldots,m$ and $m \geq 1$. By Holder inequality for sequences and Minkowski inequality for the L_α-norm, $0 < \alpha < 1$, we get

$$E[\phi(\underline{X}(t))] \leq \sum_{i=1}^{m} E[\phi_i^{\alpha}(\underline{X}(\alpha t))]\, E[\phi_i^{1-\alpha}(\underline{X}((1-\alpha)t))]$$

$$\leq \Big(\sum_{i=1}^{m} [E[\phi_i^{\alpha}(\underline{X}(\alpha t))]]^{1/\alpha}\Big)^{\alpha} \Big(\sum_{i=1}^{m} [E[\phi_i^{1-\alpha}(\underline{X}((1-\alpha)t))]]^{1/1-\alpha}\Big)^{1-\alpha}$$

$$\leq E[\phi^{\alpha}(\underline{X}(\alpha t))]\, E[\phi^{1-\alpha}(\underline{X}((1-\alpha)t))] .$$

The general result now follows by taking monotone sequences of such functions and passing to the limit.

Remark 3.5. In a recent paper by Marshall and Shaked (1982) the authors established a similar characterization for their class of multivariate NBU distributions. However they used a different method to extend the validity of the inequality in (3.7) from indicator functions to nonnegative linear combinations of indicator functions.

The second characterization for the MNBU class, which is useful in relating it to a multivariate concept of NBU distributions, follows from the following lemma which is due to Block and Savits (1980). We provide here a simple and new proof.

Lemma 3.6. Let $U \subseteq R^n$ be an open upper set. Then $U = \bigcup_{i=1}^{\infty} Q_{\underline{x}^i}$, where $\underline{x}^i \in R^n$, $i = 1,2,\ldots$.

Proof. Let $\underline{x}$ be an arbitrary element in U. Since U is open there exists a $\underline{y} \in U$ such that $\underline{y} < \underline{x}$ and y_i is rational, i = 1,2,..., n. The result now follows by observing that $\underline{x} \in Q_{\underline{y}} \subseteq U$ and that the set of points in R^n with rational coordinates is countable.

Theorem 3.7. The following condition is equivalent to (3.5),

$$P(\underline{X}(t) \in U) \leq P(\underline{X}(\alpha t) \in U)P(\underline{X}((1-\alpha)t) \in U),$$

for all $t \geq 0$, $0 \leq \alpha \leq 1$ and all U of the form $\bigcup_{i=1}^{p} Q_{\underline{x}^i}$ where $\underline{x}^i \in R^n$, i = 1,...,p, p is a positive integer.

Proof. Obviously (3.5) implies (3.8). Now assume (3.8) is true and let U be an open upper set. By lemma 3.6 there exists a sequence $U_1 \subseteq U_2 \subseteq \ldots$ increasing to U and (3.8) is true for each U_i. The result now follows by passing to the limit.

4. Closure Properties, Examples and a Related Class of Multivariate NBU Distributions.

Closure properties of the MNBU class, which correspond to common reliability operations, are established in this section. Such properties are then utilized to construct and identify members of the MNBU class.

In the following theorem three fundamental closure properties of the MNBU class are established.

Theorem 4.1. The following properties hold for the MNBU class:
(P Let $\underline{X}(t) = (X_1(t),\ldots,X_n(t)): t \geq 0\}$ be a MNBU process and let $f:R^n \to R^m$ be a nonnegative, nondecreasing and left-continuous function. Then $\{f(\underline{X}(t)): t \geq 0\}$ is a MNBU process.

(P2) Let $\{\underline{X}(t) = (X_1(t),\ldots,X_n(t)): t \geq 0\}$ and $\{\underline{Y}(t) = (Y_1(t),\ldots,Y_m(t)): t$ be two independent MNBU processes. Then $\{(\underline{X}(t); \underline{Y}(t)): t \geq 0\}$ is a $(m + n)$ - dimensional MNBU process.

(P3) Let $\{\underline{X}_n(t): t \geq 0\}$, n=1,2,... be a sequence of k-dimensional MNBU processes and $\underline{X}_n(t) \downarrow \underline{X}(t)$ for every t, where $\{\underline{X}(t): t \geq 0\}$ is nonnegative nonincreasing and right-continuous process. Then $\{\underline{X}(t): t \geq 0\}$ is a MNBU process.

Proof. (P1) The proof is obvious in view of theorem (3.4).

(P2) The proof is similar to the one given by Marshall and Shaked (1982), pages 202-203, to establish a similar property for their class of multivariate NBU distributions. The details are therefore omitted.

(P3) Let $U \subseteq R^k$ be an open upper set. Let $\phi_m: R^k \to R$ be a sequence of continuous, bounded an nondecreasing functions such that $\phi_m \to I_U$ pointwise. By theorem 3.4 we have

$$E[\phi_m(\underline{X}(t))] \leq E[\phi_m^{\alpha}(\underline{X}_n(\alpha t))]\ E[\phi_m^{1-\alpha}(\underline{X}_i((1-\alpha)))]\ ,$$

The result now follows by taking limits as $n \to \infty$ then as $m \to \infty$.

Remark 4.2. The assumption of left-continuity in (P1) is not needed when the state space for the process $\{\underline{X}(t): t \geq 0\}$ is finite.

The folowing properties for the MNBU class are immediate consequences of the fundamental properties (P1) and (P2).

Corollary 4.3. The following properties hold for the MNBU class:

(P3) Le $\{X_i(t): t \geq 0\}$, i=1,...,n be independent univariate NBU processes. Then $\{\underline{X}(t) = (X_1(t),\ldots,X_n(t)): t \geq 0\}$ is a MNBU process.

(P4) Let $\{\underline{X}(t): t \geq 0\}$ be a n-dimesional MNBU process. Then $\{X(t) = (X_{i_1}(t),\ldots,X_{i_k}(t)): t \geq 0\}$ is a k-dimensional MNBU process, where $1 \leq k \leq n$ and $1 \leq i_1 < \ldots < i_k \leq n$.

(P5) Let $\{\underline{X}(t) = (X_1(t),\ldots,X_n(t)): t \geq 0\}$ be a MNBU process and let $\phi_i:R^n \to R$ be nonnegative, nondecreasing and left-continuous function, $i=1,\ldots,m$. Then $\{(\phi_1(\underline{X}(t)),\ldots,\phi_m(\underline{X}(t)): t \geq 0\}$ is a MNBU process.

(P6) Let $\{\underline{X}_i(\underline{t}): t \geq 0\}$, $i=1,..,k$ be k independent MNBU processes of the same dimension. Then $\{\sum_{i=1}^{k} \underline{X}_i(t): t \geq 0\}$ is a MNBU process.

Proof. The proof is obvious and is therefore omitted.

Remark 4.4. It should be noted that the generalized NBU closure theorem due to El-Neweihi, Proschan and Sethuraman (1978) is a special case of (P5). Also observe that (P6) asserts the closure of the MNBU class under convolution.

The following theorem relates the MNBU class to a well known class of multivariate NBU random vectors, namely the class $C=\{\underline{T}=(T_1,\ldots,T_n)$: $\underline{T}$ is nonnegative and $\tau(\underline{T})$ is NBU for every monotone life function τ, n is arbitrary positive integer . (recall that $\tau(\underline{T})$ has the form $\max_{1 \leq j \leq p} \min_{i \in A_j} T_i$, where $\bigcup_{j=1}^{p} A_j \subseteq \{1,\ldots,n\}\}$.

Theorem 4.5. The process $\{\underline{X}(t): t \geq 0\}$ is a MNBU process if and only if every finite collection of $\{T_{ix}: 1 \leq i \leq n, x \in R\}$ is in C, where $T_{ix} = \inf\{t: X_i(t) \leq x\}$.

Proof. The proof follows readily by theorem 3.7 and the simple observation that for any two upper Borel sets U_1, U_2, $T_{U_1 \cup U_2} = \max(T_{U_1}, T_{U_2})$ and $T_{U_1 \cap U_2} = \min(T_{U_1}, T_{U_2})$. The simple details of the proof are left to the reader.

Remark 4.6. The above theorem is essentially the analogue of theorem 2.4 of Block and Savits (1981).

The above properties of the MMBU class and its relationship to the class C of multivariate MBU random vectors can be utilized to construct many examples of MMBU processes. For instance if $\underline{T}$ belongs to C then $\{(I_{(T_1 > t)},\ldots,I_{(T_n > t)}): t > 0\}$ is a MMBU process. Also multivariate processes whose coordinates are monotone functions of independent univariate MBU processes are MMBU processes. The following example motivated by the theory of multistate coherent systems illustrates the construction and identification of members of the MMBU class.

Example 4.7. Consider a system of n binary independent components. Assume we have M-1 spares for each of the n components. A failed component is instantaneously replaced by one of its spares. All the lifelengths are assumed to be independent MBU random variables. When the original component i is functioning (and none of the spares has been used), we consider that component i is in state M. Upon failure of the original component one of its spares is used and so the component now enters state M-1, etc. Let $\{\underline{X}(t) = (X_1(t),\ldots,X_n(t)): t \geq 0\}$ be the process describing the states of the n components as time passes. Then $\{\underline{X}(t): t \geq 0\}$ is a MMBU process. Moreover if the n components are forming a coherent system whose cut sets are $C_1,\ldots C_r$ and $Y_j(t) = \max_{i \in C_j} (X_i(t))$, j=1,...,r, then $\{(Y_1(t),\ldots,Y_r(t)): t \geq 0\}$ is also a MMBU process.

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